B.Tech. DEGREE EXAMINATION, MAY 2016
Second Semester

15MA102 – ADVANCED CALCULUS AND COMPLEX ANALYSIS
(For the candidates admitted during the academic year 2015 -2016)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. $\int_1^b\int_1^a \frac{dxdy}{xy}$ is equal to
   (A) log a +log b
   (B) log a
   (C) log b
   (D) log a log b

2. The value of $\int_0^1\int_0^2\int_0^3 dxdydz$ is
   (A) 3
   (B) 4
   (C) 2
   (D) 6

3. Area of the double integral in polar co-ordinate is equal to
   (A) $\iint_R drd\theta$
   (B) $\iint_R r^2drd\theta$
   (C) $\iint_R rdrd\theta$
   (D) $\iint_R (r+1)drd\theta$

4. What is the value of $\int_0^1\int_0^x dxdy$
   (A) $\frac{1}{2}$
   (B) -1
   (C) 1
   (D) $\frac{1}{3}$

5. Curl (grad φ) is
   (A) -1
   (B) 1
   (C) 0
   (D) φ

6. If $\varphi = xyz$ then $\nabla\phi$ is
   (A) $yz\vec{i} + zx\vec{j} + xy\vec{k}$
   (B) $xy\vec{i} + yz\vec{j} + zx\vec{k}$
   (C) 0
   (D) $zx\vec{i} + xy\vec{j} + yz\vec{k}$

7. The condition for $\bar{F}$ to be conservative is $\bar{F}$ should be
   (A) Solenoidal vector
   (B) Irrotational vector
   (C) Rotational vector
   (D) Neither solenidal nor irrotational

8. The connection between a line integral and double integral is known as
(A) Green's theorem
(B) Stoke's theorem
(C) Divergence theorem
(D) Convolution theorem

9. L (sin 3t) is equal to
(A) $\frac{3}{s^2-3}$
(B) $\frac{s}{s^2-3}$
(C) $\frac{3}{s^2+9}$
(D) $\frac{s}{s^2+9}$

10. The value of the function $f(t) = ae^{-bt}$ using initial value theorem is
(A) a
(B) a^2
(C) ab
(D) 0

11. $L\left[\int_0^t \sin t\, dt\right] =$ ______________
(A) $\frac{1}{s^2+1}$
(B) $\frac{s}{s^2+1}$
(C) $\frac{1}{s(s^2+1)}$
(D) $\frac{1}{(s^2+1)^2}$

12. L (t^4) is equal to
(A) $\frac{3!}{s^4}$
(B) $\frac{4!}{s^4}$
(C) $\frac{4!}{s^5}$
(D) $\frac{5!}{s^4}$

13. An analytic function with constant modulus is
(A) A function of x
(B) A function of y
(C) Constant
(D) A function of z

14. If u +iv is analytic, then the curves $u = c_1$ and $v = c_2$.
(A) Intersect each other
(B) Cut orthogonally
(C) Are parallel
(D) Coincides

15. The function $w = \sin x \cosh y + i \cos x \sinh y$ is
(A) Continuous
(B) Differentiate at origin
(C) Analytic
(D) Need not be analytic

16. The invariant points of the transformation $W = \frac{z-1}{z+1}$ is
(A) $\pm i$
(B) ± 1
(C) 2,7
(D) 0

17. The value of $\oint_c \frac{(3z^2+7z+1)}{z+1} dz$ where C is $|z| = \frac{1}{2}$ is equal to
(A) $2\pi i$
(B) 0
(C) πi
(D) -1

18. The singularity of $f(z)=\frac{z}{(z-2)^3}$ is

(A) Essential singularity
(B) Removable singularity
(C) Pole of order 3
(D) Pole of order 1

19. If f(z) is analytic inside and on C, the value of $\oint_c \frac{f(z)}{(z-a)^2}dz$, where C is a simple closed curve and 'a' is any point with in 'C' is

(A) $f^1(a)$
(B) 0
(C) $\pi i f^1(a)$
(D) $2\pi i f^1(a)$

20. The part $\sum_{n=1}^{\infty} b_n(z-a)^{-n}$ is consisting of negative integral powers of (z-a) is called as

(A) Analytic part of Laurent series
(B) The principal part of Laurent series
(C) Real part of Laurent series
(D) Imaginary part of Laurent series

PART – B ($5 \times 4 = 20$ Marks)

Answer **ANY FIVE** Questions

21. Using double integration, find the area of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

22. Show that $r^n\bar{r}$ is an irrotational vector for any value of 'n' and is solenoidal for $n=-3$.

23. Find $L\left[\frac{e^{-t}-e^{-3t}}{t}\right]$.

24. Verify final value theorem for the function $1+e^{-t}(\sin t+\cos t)$.

25. Find the constants a, b, c if $f(z)=x+ay+i(bx+cy)$ is analytic.

26. Evaluate using divergence theorem $\iint_s (x+z)dydz+(y+z)dzdx+(x+y)dxdy$ over the sphere $x^2+y^2+z^2=4$.

27. Evaluate $\oint_C \frac{(z+1)}{z(z-1)}dz; C \text{ is } |z|=2$.

PART – C ($5 \times 12 = 60$ Marks)

Answer **ALL** Questions

28. a. Change the order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$.

(OR)

b. Find the volume of the sphere $x^2 + y^2 + z^2 = a^2$ by using triple integration.

29. a. Verify Stoke's theorem for $\bar{F} = (y - z + 2)\bar{i} + (yz + 4)\bar{j} - xy\bar{k}$ where S is an open surface of a cube x =0, x =2, y =0, y =2 and z =0, z =2.

(OR)

b.i. Show that $\bar{A} = (x^2 + y^2 + x)\hat{i} + (2xy + y)\hat{j}$ is irrotational and hence find the scalar potential.

ii. Evaluate $\oint_C (x^2 + y^2)dx - 2xydy$ taken over the rectangle bounded by the lines x =±a, y =0, y =b.

30. a.i. Find the Laplace transform of $f(t) = \begin{cases} t & , 0 < t \le 2 \\ 4 - t, & 2 \le t < 4 \end{cases}$ and satisfy $f(t + 4) = f(t)$.

ii. Find $L[t e^{-t} \sin t]$.

(OR)

b. Using Laplace transform method solve $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^{-t}$ given x (0) =2, x' (0) =1.

31. a. Find the analytic function $f(z) = u + iv$ where $u - v = \frac{\sin 2x}{\cosh 2y - \cos 2x}$.

(OR)

b. Find the bilinear transformation that maps the point ∞, i, 0 into 0, i, ∞ respectively.

32. a.i. Evaluate $\oint_C \frac{e^{2z}}{\cos \pi z} dz$ where C is a circle $|z| = 1$.

ii. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ as a Laurent's series valid in the region.

(1) $|z| < 1$
(2) $1 < |z| < 2$
(3) $|z| > 2$.

(OR)

b. Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ by contour integration.

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